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How F.G.W. Struve started his ambitious project?

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1. Introduction

In the history of science F.G.W. Struve is known above all as a great astronomer and a promotor of astronomical science. However, he has made a serious contribution to geodesy as well – among his 129 works 48 are devoted to geodesy. In this field he will be mostly remembered as an undertaker of an extremely long meridian arc measurement. In the following report we will discuss how Struve started this ambitious project and how he implemented the important preceding project of the triangulation of Livonia.
This report is mostly based on the book by Struve (1844) and the papers by Zhelnin (1964), Vuuk (1968), Zhelnin and Vallner (1969) and Torim (1994).

2. Some historical remarks

We know that Struve came to Tartu University as a refugee from Altona to study literature in 1808 as his father had recommended. Indeed, he graduated from the university with a gold medal at the end of 1810, presenting his essay "De studio critices et grammatices apud Alexandrinos" which is unfortunately not preserved. Since Rector Magnificus professor Parrot offered him a stipend of 500 roubles he declined the offer to become a senior teacher of history in Tartu Gymnasium and he decided to further his studies in mathematics and physics. Already in 1813 Struve passed the qualifying exam for the master's and doctor's degree and presented the thesis "De geographica speculae

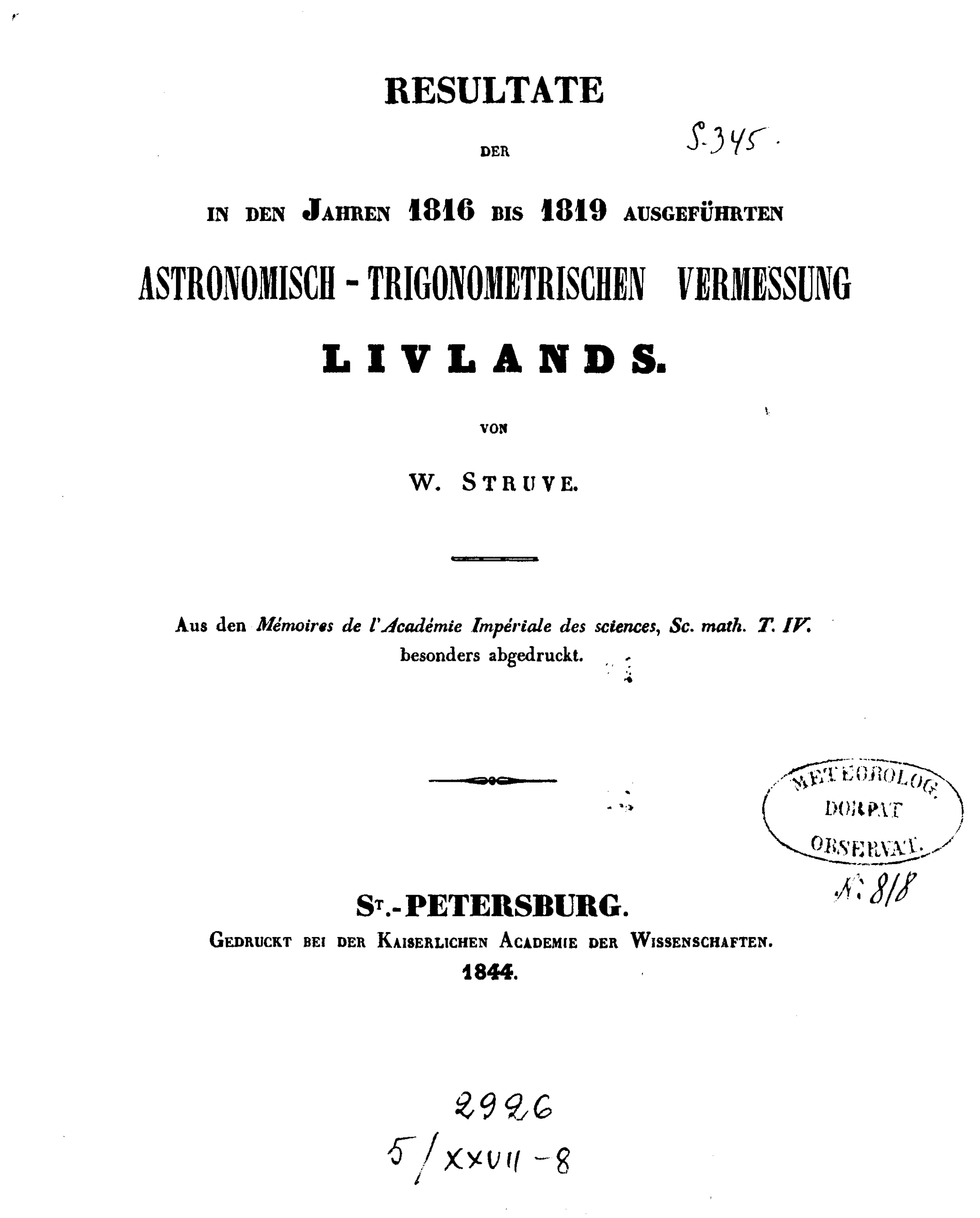
Dorpatensis positione”. This was not his first work in geodesy. In 1812 when he was spending his summer in Sangaste (Sagnitz) manor he was practising land survey with a 10-inch Troughton sextant which he had bought for his meagre savings. He was arrested by a Russian patrol – one of the Russian Army corps had been brought to Livonia in order to defend the avenue of approach to St Petersburg against the French army. He was taken for a French spy and brought to Pärnu where he had to appear before a military court where the misunderstanding was quickly cleared but Struve lost a week's work.

Around 1815 the Livonian Society of Public Utility and Economy (Livländische oeconomische und gemeinnützige Societät) applied to the Tartu University for technical help in land survey. They planned to issue a topographic map of Livonia (now the southern part of Estonia and the northern part of Latvia – approximately 44000 square kilometers between the latitudes of 56 degrees 32 minutes and 59 degrees). The university made a contract with Struve who was then the extraordinary professor of mathematics and astronomy. He was allowed to spend only three to four summer months for this work. The contract foresaw that all the expenses would be covered by the Society – approximately 3000 silver roubles, as planned. Struve has been even given a horse and a waggon (Struve, 1844)!

3. Instruments

There were two sets of instruments - as a main instrument for measuring the horizontal angles, azimuths and latitudes Struve used a 10-inch Troughton mirror sextant which was given him by the Society. Besides of that Struve used a 2' telescope to find the far-away trigonometric marks, Arnold's pocket chronometer, Baumann's artificial horizon and a horizontal sector (with the objective diameter of 50 mm, focal length of 480 mm and magnification of 30X) to measure vertical angles. It allowed to measure vertical angles up and down to 10 degrees with the accuracy of 4 arcseconds. This instrument was constructed by Struve himself and made in the workshop of the university. In the beginning Struve used an elevation measurer – “Höheninstrument” borrowed from prof. Moritz Engelhardt but evidently this was not accurate enough.

The second set consisted of a smaller sextant, artificial horizon, made of thick black polished glass and a pocket-watch, able to measure seconds. Carl Friedrich Knorre, the son of the former extraordinary professor of astronomy in Tartu University Ernst Knorre and later to become the astronomer of the Imperial Black Sea Navy (Levitskij, 1899),



RESULTATE

DER

IN DEN JAHREN 1816 BIS 1819 AUSGEFÜHRTEN

ASTRONOMISCH - TRIGONOMETRISCHEN VERMESSUNG

LIVLANDS.

VON

W. STRUVE.

Aus den *Mémoires de l'Académie Impériale des sciences, Sc. math. T. IV.* besonders abgedruckt.

ST.-PETERSBURG.

GEDRUCKT BEI DER KAISERLICHEN ACADEMIE DER WISSENSCHAFTEN.

1844.

Fig.1. The title page of the book about the Livlands triangulation

operated the second set (Vuuk, 1968).

To measure the baseline he had designed an instrument consisting of five wooden rods, each 1 toise long (one toise is 1.949 m), they were compared against an iron rod in the university observatory which was one toise long and which was certified in Paris on April 6, 1784 at 14 degrees R. Two identical sets of rods were made, each set was connected with pin joints, making the total length of 9.75 m. These rods were made of fir-tree, boiled

in linseed varnish and lacquered. The terminal rods had silvered scales.
This set of instruments was hardly appropriate for high-accuracy measurements.

4. The baseline measurements

For the measuring of the baseline in Februar 1819 Struve decided to use the flat surface of ice on lake Võrtsjärv. This elegant approach was not new – it has been used by Joseph Delisle in 1737 when he started to measure the meridian arc through St Petersburg (this work was never completed). The triangulation network was constructed on the 13.34 km baseline between villages Uniküla and Rannaküla, both on the shore. The base was measured only once and in one direction – there was no time to make two passages. The temperature was taken into account, the thermal expansion of iron was known and the expansion of fir-tree was measured experimentally.
The base-line of 600 toise = 1169 m for the triangulation in the western part of Livonia was measured with a steel chain along the ice of river Daugava (Düna) at the end of the winter of 1818 (this measurement was done by a headmaster of one of the schools in Riga – Wilhelm Friedrich Keussler) . A similar instrument was used to measure the baseline for the Pärnu triangulation (it is not clear whether this was the same chain which was used in Riga). This time the measurement took place not on the ice but along the flat coastal meadow near Pärnu and it was found to be 2158.43 m. The steel chain was repeatedly checked before and after the measurements along the brass ruler on the wooden floor of the Audru church at the temperature of 15.75 degrees R.
For the astronomical source of the geodetic network Struve chose the university observatory whose geographical coordinates he had measured for his doctoral thesis, and the cathedral in Riga.

5. Triangulation in Eastern and Southern Livonia

In planning his activities Struve relied on his excursions in the countryside when he was tutoring the children of Count von Berg at Sangaste. The landscape in these parts was (and still is) rather open so Struve chose as main triangle points the local buildings. The list includes 2 observatories, 74 church spires, 73 windmills, 1 lighthouse, 113 other buildings, like factories, manor houses, etc. There were also 6 single fir-trees. Altogether 292 points were used, including 63 geodetic signals and marks which were built during the triangulation. None of them is preserved.

Resultate der astronom.-trigonometr. Vermessung Livlands. 23

	Name des Punkts	Abscisse	Ordinate	Abscisse	Ordinate	Breite	Länge von Ferro
		in Toisen		in Saschen			
1.	Kreutzburg Kirchthurm	0	+ 13604	0	+ 12427	56°30′48″,7	43°31′18″,9
2.	Daborskaln Signal	4058	+ 8428	3707	+ 7699	35 6,1	43 21 30,9
3.	Stockmannshof Schlossthurm..	4875	+ 6780	4453	+ 6194	35 57,9	43 18 23,0
4,	Grütershof Begräbnisskapelle .	5117	+ 3687	4674	+ 3368	36 13,6	43 12 29,7
5.	Kockenhusen Wohnhaus	7379	— 59	6741	— 54	38 36,3	43 5 21,8
6.	Alt-Bewershof Windmühle (*h*)	11966	+ 1752	10931	+ 1600	43 25,4	43 8 49,2
7.	Monbijou Thurm	12050	+ 17838	11008	+ 16295	43 25,9	43 39 32,9
8.	Alt-Kroppenhof Wohnhaus ...	12089	— 8746	11043	— 7990	43 32,0	42 48 46,1
9.	Fehteln Kirchthurm	12215	+ 10440	11158	+ 9537	43.39,4	43.25 25,1
10.	Kroppenhof Kirchthurm	12745	— 9480	11643	— 8660	44 13,1	42 47 21,6
11.	Mahrzen Wohnhaus	14938	+ 21185	13646	+ 19353	56 46 26,0	43 45 59,8
12.	Kampeskaln Signal	15718	+ 18066	14359	+ 16504	47 16,9	43 40 2,7
13.	Saadsen Windmühle (*st*)	16069	— 9437	14679	— 8621	47 42,6	42 47 24,8
14.	Saadsen Wohnhaus	16129	— 9613	14734	— 8782	47 46,3	42 47 4,6
15.	Bersohn Kirchthurm	16763	+ 18978	15313	+ 17337	48 22,3	43 41 48,4
16.	Grosdohn Wohnhaus	17506	+ 19659	15992	+ 17959	49 8,7	43 43 7,4
17.	Gross-Oselhof Windmühle(*h*)..	17577	+ 7086	16057	+ 6473	49 18,1	43 19 2,7
18.	Linden Pastorat	17645	+ 4401	16119	+ 4020	49 22,9	43 13 54,2
19.	Grosdohn Windmühle (*h*) ...	17664	+ 19634	16136	+ 17936	49 18,7	43 43 4,7
20.	Praulen Wohnhaus	17984	+ 27565	16429	+ 25181	49 33,3	43 58 16,5
21.	Neu-Lasdohn Wohnhaus	17999	+ 25993	16442	+ 23745	56 49 35 5	43 55 15,9
22.	Alt-Lasdohn Windmühle (*h*) ..	18011	+ 25750	16453	+ 23523	49 36,4	43 54 48,0
23.	Spirekaln Signal	18065	+ 13885	16503	+ 12684	49 46,8	43 32 4,4
24.	Essen Windmühle (*h*)	18462	— 9273	16865	— 8471	50 13,4	42 47 42,5
25	Minnaberg Standpunkt	18590	+ 26803	16982	+ 24485	50 12,0	43 56 49,8
26.	Sestukaln Signal	18660	+ 6636	17046	+ 6062	50 26,4	43 18 11,5
27.	Lasdohn Kirchthurm	18752	+ 25643	17130	+ 23425	50 23,2	43 54 36,7
28	Praulen Begräbnisskapelle	18847	+ 26804	17217	+ 24486	50 28,3	43 56 50,3
29.	Festen Kirchthurm	19387	+ 13285	17710	+ 12136	51 10,4	43 30 56,4
30.	Gaisekaln Signal	20427	+ 16726	18660	+ 15280	52 14,3	43 37 33,1

Fig.2. The extract of the Struve's results

The triangulation network was divided into three categories according to the accuracy. In the first category of 90 triangles only for 53 all three angles were measured. For remained 37 only two angles were measured and the third was assumed to be π – the sum of the other two angles. For the triangles of the second category mostly two angles were

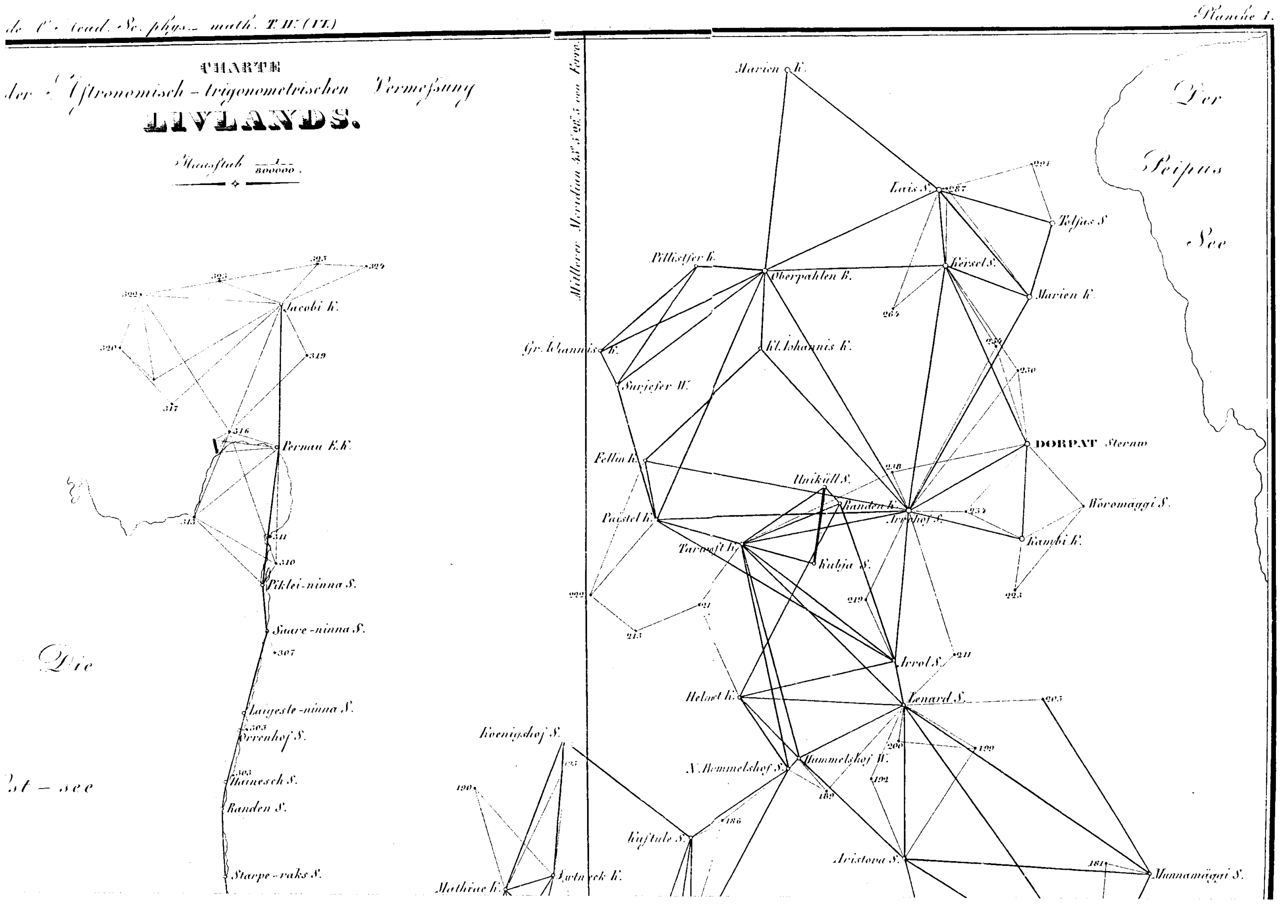


Fig.3. The northern part of the Livlands triangulation map

measured. The average length of the sides of the triangles was around 25 km – the longest was 60 km.

6. Astronomical-trigonometric survey from Riga to Pärnu

The landscape in this part of Livonia was (and still is) rather closed. This prevented Struve to use the approach he had so successfully exploited in eastern and southern parts. So he established a chain of astronomical points from Riga to Pärnu where he measured the azimuths and partly also geographical latitudes. This method, later developed further by Struve himself, was to become known as parallactic polygonometry. The traverse between Riga and Pärnu ran mostly along the meridian and its length is 186 km. This network was connected with the inland triangulation with only one point (Cathedral in Riga) thus making the accuracy of the whole project much worse.

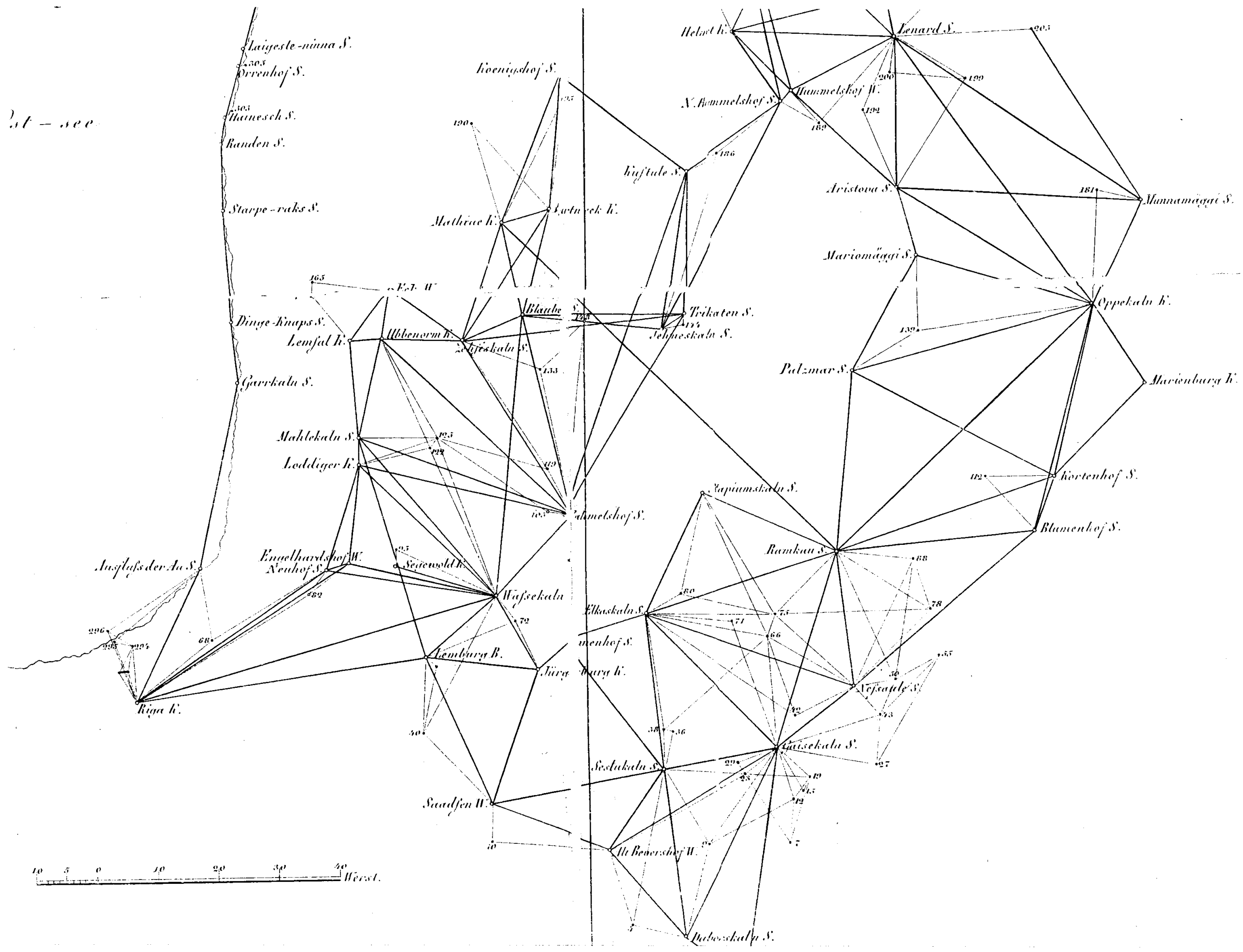


Fig.4. The southern part of the Livlands triangulation map

7. Trigonometric measurement of elevation

In measuring the elevations above the sea level Struve used the method of trigonometric levelling. Since he had neither a Borda circle nor Ramsden nor Reichenbach theodolites he had to cope first with the Engelhardt's “elevation measurer” and later with the horizontal sector. The elevations were measured altogether in 280 points, so some of the points were left unmeasured. According to Vuuk (1968) the zero point for his elevation measurement was taken the average sea level in the estuary of river Daugava (at that time the systematic zero level was not introduced – only in 1870s the Kronstadt zero point was defined). Using this data Struve was able to calculate the refraction coefficient of the Earth atmosphere which he found to be 0.2137, thus only slightly less than the result by Gauss – 0.3106. Both of these results differed seriously from that by Delambre – 0.1678.

8. Accuracy of triangulation

As a genuine representative of the exact sciences Struve always considered the possible errors during the measurements. First of all you have to know the accuracy of the instruments available. In three consecutive years he checked his main instrument - the Troughton sextant – by measuring horizontal angles over the horizon. He found the errors to be in the range from 2'53” to 3'21”. Since he did not reduce the horizontal angle measurements to true horizon he made an error which according to Estonian geodesist Vuuk (1968) was up to 3”. For quite a few points the angles were measured eccentrically, the instrument being shifted up to 4.5 m. Due to the fact that he could use only the summer months free of lectures he had to make measurements even at 10:00 in July mornings which definitely did not add to accuracy. A. Vuuk has estimated the mean square errors in measurement of angles and found them to be 15 arcseconds.

The length of the main baseline on the ice of the lake Võrtsjärv was not reduced to the sea-level because Struve assumed the respective error to be very small. He estimated the accuracy of the baseline to be 1:40000. Nothing is said about the accuracy in baseline measurements in Riga and Audru.

Vuuk (1968) has estimated also the mean square errors in levelling. He found that they are rather large – reaching 6 m at 50 km base.

9. Summary

The triangulation of Livonia was meant as a basic network for compiling a map of Livonia. The map was to be drawn up by a Livonian nobleman Carl Gottlieb Rücker who was the adjured surveyor of the Livonian Knighthood. Due to some resistance of the local landowners the map was ready only in 1839 – 23 years after Struve finished his triangulation. F.G.W. Struve intended to remeasure at least a part of the Livonian triangulation network but instead of that he started a much bigger project – measuring the meridian arc of 25 degrees 20 minutes – which can be considered as one part of his lifework.

In conclusion we may say that the triangulation of Livonia – a big and exacting task for young and unexperienced Struve - was carried out with flying colours. The experience accumulated during this work laid a solid basis for an enormous future project – measuring the big arc.

10. Literature

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